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ARMY MISSILE RESEARCH, DEVELOPMENT AND ENGINEERING LABORATORY, REDSTONE ARSENAL, ALABAMA

30 DECEMBER 1976

TECHNICAL REPORT RG-77-5

TRANSIENTS ON A LOSSLESS, EXPONENTIALLY-TAPERED TRANSMISSION LINE

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30 December 1976

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U.S. ARMY MISSILE COMMAND
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A solution for the problem of a voltage pulse propagating along a lossless, exponentially-tapered transmission line is presented in the form of a convolution integral. If the input pulse can be approximated in terms of simple functions, an analytical solution is possible. If not, numerical evaluation of the resulting integral is still much simpler than solving the differential equations of propagation.

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TRANSIENTS ON A LOSSLESS, EXPONENTIALLY-TAPERED TRANSMISSION LINE

The following is an analytical solution for a voltage pulse travelling along an exponentially-tapered transmission line in which series and shunt resistances may be neglected. It is valid up to the point of reflection at the end of the line and is thus the solution for a "semi-infinite" transmission line with an arbitrary input pulse. It may be used as a very quick check of more general computer solutions for nonuniform transmission line problems.[1,2]

Figure 1 shows the line schematically and defines symbols. Inductance and capacitance along the line are assumed to vary as

$$L(x) = L_0 e^{\alpha x} \tag{1}$$

$$C(x) = C_0 e^{-\alpha x} \quad . \tag{2}$$

The equations to be solved are then

$$\frac{\partial}{\partial x} V(x,t) + L(x) \frac{\partial}{\partial t} I(x,t) = 0 \tag{3}$$

$$C(x) \frac{\partial}{\partial t} V(x,t) + \frac{\partial}{\partial x} I(x,t) = 0 \quad . \tag{4}$$

Using Laplace transforms and boundary conditions $I(x,0) = V(x,0) = 0$, the resulting equations may be uncoupled to yield

$$\frac{d^2}{dx^2} \overline{V}(x,s) - \frac{1}{L}\frac{dL}{dx}\frac{d}{dx} \overline{V}(x,s) - s^2 LC\overline{V}(x,s) = 0 \tag{5}$$

$$\frac{d^2}{dx^2} \overline{I}(x,s) - \frac{1}{C}\frac{dC}{dx}\frac{d}{dx} \overline{I}(x,s) - s^2 LC\overline{I}(x,s) = 0 \tag{6}$$

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[1] Hill, J. L. and Mathews, David, Transient Analysis of Systems With Exponential Transmission Lines (to be published).

[2] Coates, B. A. and Butler, Chalmers M., Transients on a Non-Uniform Transmission Line, Bulletin No. 11, University of Mississippi Engineering Experiment Station, University, Mississippi, May 1969.

or, considering only the voltage, Equation (5) becomes

$$\frac{d^2\overline{V}}{dx^2} - \alpha \frac{d\overline{V}}{dx} - \frac{s^2}{c_p^2}\overline{V} = 0 \tag{7}$$

in which $c_p = 1/\sqrt{L(x)C(x)} = 1/\sqrt{L_0C_0}$ is the speed of propagation of the wavefront. The differential equation for the voltage has the general solution

$$\overline{V}(x,s) = e^{\alpha x/2}\left[A(s)e^{k\sqrt{s^2+\beta^2}} + B(s)e^{-k\sqrt{s^2+\beta^2}}\right] \tag{8}$$

in which

$$\beta^2 = \frac{\alpha^2 c_p^2}{4} \tag{9}$$

$$k = \frac{x}{c_p} \quad .$$

For the case of no reflections, $A(s) = 0$ in Equation (8). At the input to the transmission line,

$$\overline{V}(0,s) = B(s) = \overline{v}(s) \quad . \tag{11}$$

The inverse transform of $v(s)\exp(-k\sqrt{s^2+\beta^2})$ is found by convolution with a known transform[3]

$$\mathcal{L}^{-1}\left\{\overline{v}(s)e^{-k\sqrt{s^2+\beta^2}}\right\} = v(t-k)\,u(t-k)$$
$$- \beta k\int_0^t \frac{v(t-\tau)J_1(\beta\sqrt{\tau^2-k^2})u(\tau-k)d\tau}{\sqrt{\tau^2-k^2}} \quad , \tag{12}$$

---

[3] Oberhettinger, F. and Badii, L., Tables of Laplace Transforms, No. 1.33, New York: Springer Publishing Co., 1973, p. 211.

in which u is the unit step function and J_1 is the first order Bessel function of the first kind, so that V(x,t) becomes

$$V(x,t) = e^{\alpha x/2}\left[v\left(t - \frac{x}{c_p}\right) u\left(t - \frac{x}{c_p}\right) - \frac{\alpha x}{2}\int_0^t \frac{v(t-\tau) J_1\left(\frac{\alpha c_p}{2}\sqrt{\tau^2 - \left(\frac{x}{c_p}\right)^2}\right) u\left(\tau - \frac{x}{c_p}\right) d\tau}{\sqrt{\tau^2 - \left(\frac{x}{c_p}\right)^2}} \right] \qquad (13)$$

or,

$$V(x,t) = \begin{cases} 0 & t \leq \frac{x}{c_p} \\ e^{\alpha x/2}\left[v\left(t - \frac{x}{c_p}\right) - \frac{x}{c_p}\int_0^{y_t} v[t - \tau(y)]\, J_1(y)\, \frac{dy}{\tau(y)} \right] & t \geq \frac{x}{c_p} \end{cases} \qquad (14)$$

in which

$$\tau(y) = \frac{2}{\alpha c_p} \sqrt{y^2 + \left(\frac{\alpha x}{2}\right)^2} \qquad (15)$$

$$y_t = \frac{\alpha c_p}{2} \sqrt{t^2 - \left(\frac{x}{c_p}\right)^2} \quad . \qquad (16)$$

The solution given in Equations (13) and (14) consists of a term representing the amplified original pulse, delayed by a time x/c_p, and a term representing the distortion of the pulse as it travels down the line. It is now possible to find V(x,t) for a given input pulse, e.g.,

$$v(t) = V_0 \left(e^{-\gamma_1 t} - e^{-\gamma_2 t}\right) \quad . \qquad (17)$$

If the given input is too complicated to evaluate analytically, Equation (14) can be evaluated numerically. This is much simpler than solving Equations (3) and (4). Results for a typical data set (Table 1) and Equation (17) are given for a particular x (Figure 2)

and a particular t (Figure 3). In obtaining the data for Figures 2 and 3, a Gaussian integration subroutine, IGRAT, was used to evaluate the integral and a general Bessel function subroutine, BESJ, to evaluate $J_1(y)$. Run time on the CDC 6600 computer for a typical problem was less than 3 sec. Figure 4 shows a simplified flow graph of the program, and Table 2 gives a listing of the main program and DESUB.

TABLE 1. DATA FOR SAMPLE CASE

$\alpha = 0.4606\ m^{-1}$	$\gamma_1 = 6 \times 10^7\ sec^{-1}$
$L_0 = 13.3$ nH	$\gamma_2 = 10^9\ sec^{-1}$
$C_0 = 833.3$ pF	air dielectric ($\epsilon_r = 1$)
$c_p = 3 \times 10^8$ m/sec	

TABLE 2. PROGRAM LISTING

```
      PROGRAM TTLINE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      COMMON/TCOM/ALF,BET1,BET2,X,T,C,BEPS
      EXTERNAL DESUB
      NAMELIST/TDAT/ALF,BET1,BET2,X,T,VØ,H,DW,
     $BEPS,EPSR,DT,FLAG,TMAX,XMAX
      XMAX=1Ø.Ø
      TMAX=3.E-8
      DT=2.E-9
      CØ=3.E8
      EPSR=1.Ø
      DW=.Ø5
      FLAG=1.
      ALF=.46Ø6
      BET1=6.E7
      BET2=1.E9
      BEPS=1.E-3
      VØ=1.
      H=.1
      X=Ø.
      T=2.666666666E-8
      READ(5,TDAT)
      WRITE(6,TDAT)
      C=CØ/SQRT(EPSR)
      NOEQ=1
      IF(FLAG.EQ.1.)GO TO 7
      TXC=X/C
      WRITE(6,6)T,TXC,TMAX
 6    FORMAT(2X,*T = *,1PE1Ø.3/3X,*TXC = *,1PE1Ø.3/4X,
     $*TMAX = *,1PE1Ø.3,/)
      GO TO 9
 7    CONTINUE
      XCT=C*T
      WRITE(6,8)X,XCT,XMAX
 8    FORMAT(2X,*X = *, 1PE1Ø.3/3X,*XCT = *,1PE1Ø.3/4X,
     $*XMAX = *,1PE1Ø.3,/)
 9    CONTINUE
      WRITE(6,5)
 5    FORMAT(1HØ,2X,*--- V(X,T) ---*,//)
    1 CONTINUE
      IF(FLAG.EQ.1.)GO TO 21
      IF(T.LT.TXC)GO TO 11
      GO TO 22
21    IF(X.GT.XCT)GO TO 11
22    CONTINUE
      CT=C*T
      WT=.5*ALF*SORT(CT*CT-X*X)
      CALL IGRAT(Ø.,WT,DW,NOEQ,DESUB,TINT)
      GO TO 12
```

TABLE 2. (Concluded)

```
11    VXT=Ø.Ø
      GO TO 14
12    CONTINUE
      VXT=VØ*EXP(ALF*X/2.)*(EXP(BET1*(X-CT)/C)-
     $EXP(BET2*(X-CT)/C)-X*TINT)
14    CONTINUE
      WRITE(6,2)X,T,VXT
      IF(FLAG.EQ.1)GO TO 3
      T=T+DT
      IF(T.LE.TMAX)GO TO 1
      GO TO 4
3     CONTINUE
      X=X+H
      IF(X.LE.XMAX)GO TO 1
4     CONTINUE
    2 FORMAT(2X,*V(*,1PE1Ø.3,*,*,1PE1Ø.3,*) = *,1PE1Ø.3)
      END
      SUBROUTINE DESUB(W,F,NOEQ)
      COMMON/TCOM/ALF,BET1,BET2,X,T,C,BEPS
      TAU=SQRT(4.*W*W/(ALF*ALF)+X*X)/C
      N=1
      CALL BESJ(W,N,BJ1,BEPS,NERR)
      F=(EXP(-BET1*(T-TAU))-EXP(-BET2*(T-TAU)))*BJ1/(C*TAU)
      RETURN
      END
```

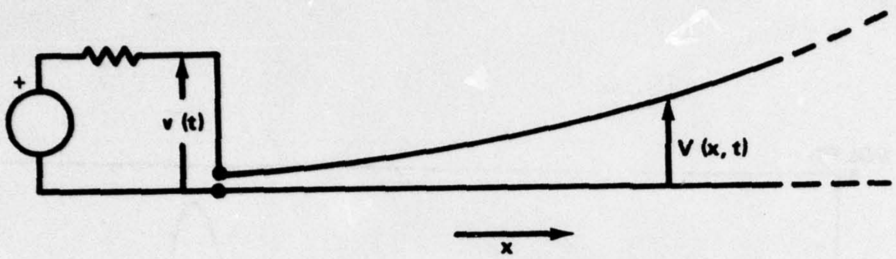


Figure 1. Schematic diagram.

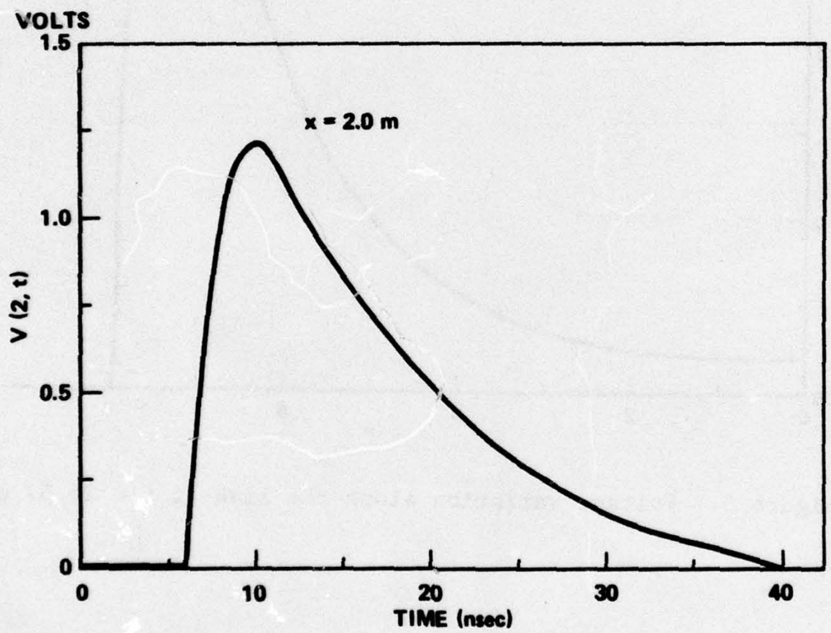


Figure 2. Voltage variation at x = 2 m from the input to the line.

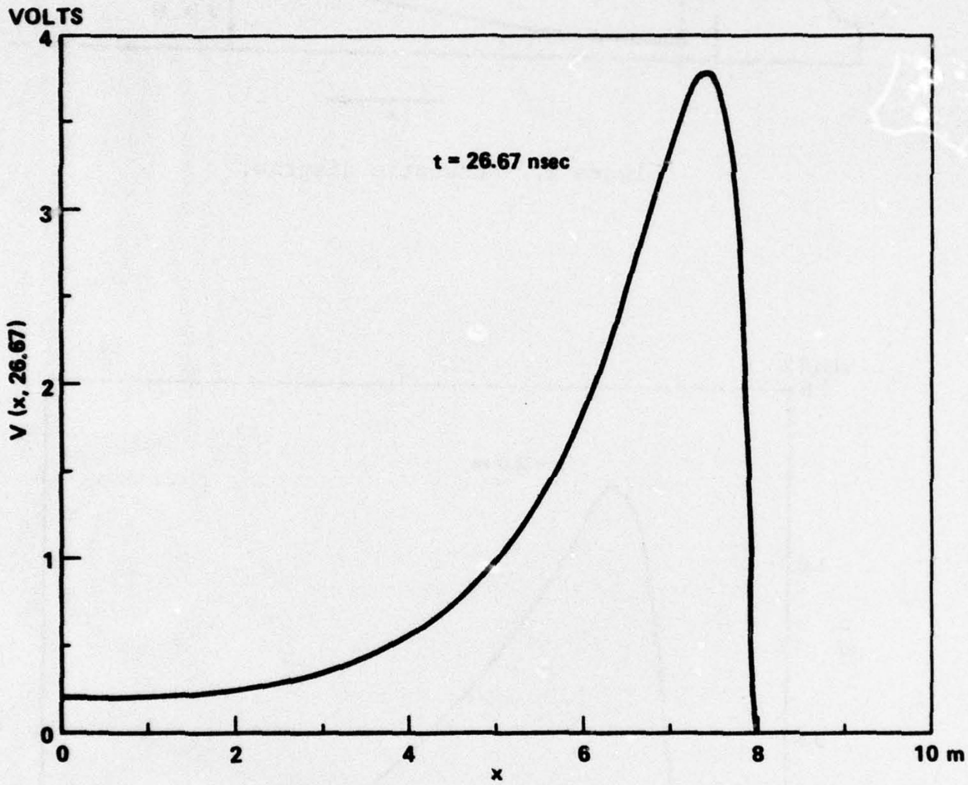
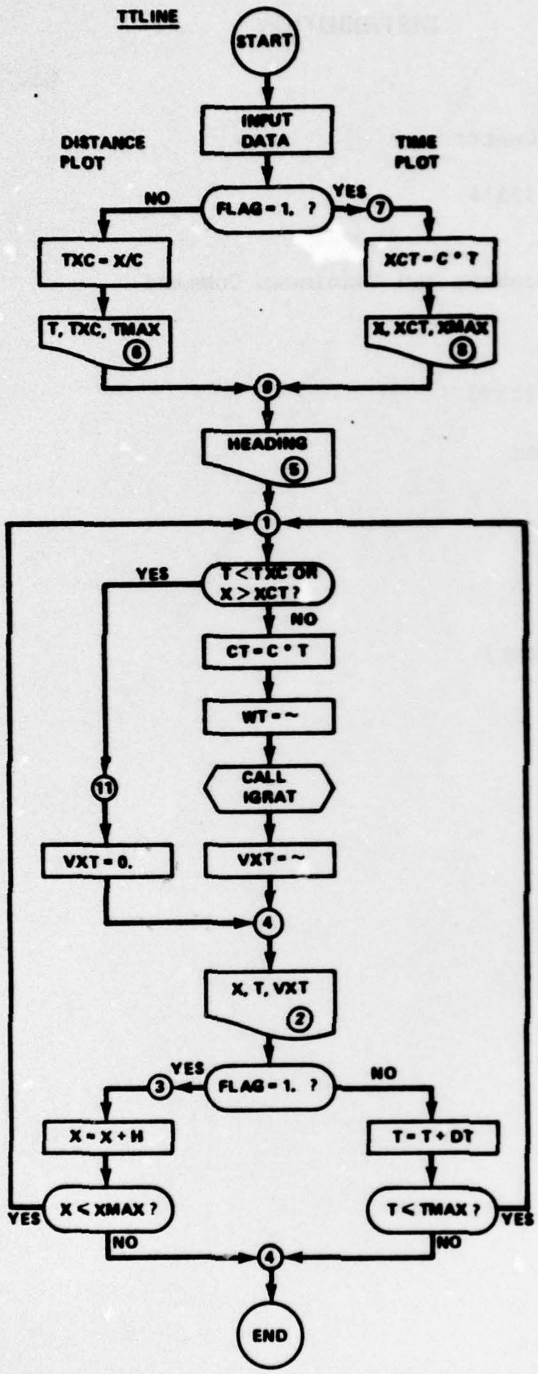


Figure 3. Voltage variation along the line at t = 26.67 nsec.



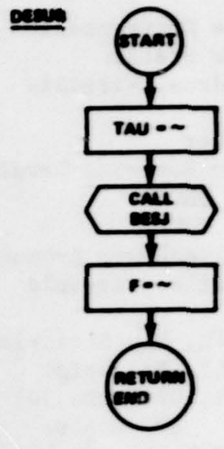


Figure 4. Flow diagram of TTLINE.